Extension of Newton's classical theory of gravitation to the Universe: new law of cosmological force as an addition to Newton's law of gravitation

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Abstract.
A new law of gravitational interaction is derived as an addition to Newton’s law of gravitation. The additional force, which follows from the new law of gravitational interaction, refers to the Universe. This force is not given by the formula of Newton’s law of gravitation. Newton’s law is applicable to describe the gravitational interaction of point masses. The new law of cosmological force is applicable to describe the gravitational interaction of bodies with the Universe beyond the applicability of Newton’s law of gravitation. The coupling constant in the law of cosmological force is the cosmological constant \( \Lambda \). The new law of cosmological force complements Newton’s law of classical gravitation. The law of cosmological force shows that in addition to the Newtonian two-body gravitational force, the gravitational force of the universe acts on all bodies. On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe it is huge and has a theoretical limit equal to the Planck force \( c^4/G \). The Newtonian force of gravitational interaction between two bodies together with the cosmological gravitational force gives the total gravitational force. The Law of universal gravitation is presented in a new form. The Law of universal gravitation contains two laws: Newton’s law of gravitation and the law of cosmological force. The Law of universal gravitation in classical representation and the quantum Law of universal gravitation are given. The law of cosmological force gives a very close force value to the Pioneer anomaly and indicates the gravitational nature of the Pioneer anomaly.

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The law of universal gravitation
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quantum theory of gravitation
large numbers
cosmological equations
parameters of the observable universe
1. Introduction

The dominant force in the universe is gravity. Newton's law of gravity was a real breakthrough in science. Newton's law of gravitation (1) impresses with its simplicity and mathematical perfection:

\[ F = G \frac{Mm}{r^2} \]  

Gravitational interaction has become the fourth fundamental interaction. Newton's law of gravitation makes it possible to explain and predict with great accuracy the motions of celestial bodies. The attractive thing about Newton's law of gravitation is the simple dependence of force on the parameters of interacting bodies.

At the same time, simple and perfect in mathematical representation, Newton's law of gravitation has limitations and limits of applicability. Newton's law of gravitation describes the interaction of two point masses. But it does not account for the gravitational interaction of bodies with the universe. It does not answer: "with what force does any mass interact with the mass of the Universe distributed in space?", "on what parameters of the Universe does the cosmological force depend?".

The possibility of application of Newton's law of gravitation ends where masses cannot be considered as point masses. There were repeated attempts to modify Newton's law and make it applicable in cosmology. In 1745 Alexis Clairaut [1] proposed a modification of Newton's law in which the law of inverse squares was changed. In 1894 Hall A. [2] proposed the replacement of the square of distance by a slightly greater degree. Hugo von Seeliger and Carl Gottfried Neumann proposed a modification of the law with a faster than Newton's law of gravitation decreasing with distance [3]. Attempts to modify Newton's law of gravitation [4] and attempts to question the law of inverse squares [3] do not stop.

No refinements and revisions of Newton's law of gravitation have made it applicable in cosmology. The simple and mathematically perfect formula of Newton's law was not applicable in cosmology. The law of inverse squares and point
masses are the main limiting factors in extending Newton's law to the Universe. Newton's law has limits of applicability. Beyond these limits it is necessary to search for another law of gravitation free from idealized point masses and the law of inverse squares.

The law of inverse squares, was formulated in 1645 by Ismail Bullialdus [3]. The law of inverse squares proved to be very productive for solving the two-body problem. This was shown by Newton's law of gravitation. The same law of inverse squares became an insurmountable obstacle in extending Newton's law of gravitation to the Universe. Obviously, with respect to the Universe there is another yet undiscovered law of gravitation, different from Newton's law of gravitation. To find a new law of gravitational interaction, we use new cosmological equations obtained by means of the law of scaling of large numbers.

2. The law of scaling of large numbers.

The coincidences of large numbers allowed us to derive the law of scaling of large numbers (Fig. 1). The law of scaling of large numbers includes two dimensionless constants: fine structure constant "alpha" and Weyl number. The law of scaling of large numbers has the form:

$$D_i = (\sqrt{\alpha D_0})^i$$

$$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8, \pm 9.$$  

Figure 1

The scaling law of large numbers. $D_0$ is a large Weyl number ($D_0 = 4.16561... \times 10^{42}$), $\alpha$ - fine structure constant

The scaling law (Fig. 1) provides a new method for calculating the values of large numbers from dimensionless constants. The scaling law generates large numbers up to scale $10^{180}$ with high accuracy. The large numbers obtained from the scaling law are close to the accuracy of the Newtonian constant of gravitation $G$. The values of the large numbers and the formulas for their calculation are given in Fig. 2.
Figure 2

Large numbers and formulas for their calculation

3. The set of coincidences of large numbers involving the cosmological constant $\Lambda$.

The table in Fig. 3 summarizes the relations of dimensional quantities containing the constant $\Lambda$.

![Table of dimensional quantities](https://example.com/table.png)

**Figure 3**

Relations of dimensional quantities containing the constant $\Lambda$. $M_0$ is the mass of the observable Universe, $\alpha$ is the fine structure constant, $\hbar$ is Planck's constant, $G$ is the Newtonian gravitational constant, $\Lambda$ is the cosmological constant, $R_0$ is the radius of the observable Universe, $T_0$ is the time of the Universe, $H$ is the Hubble constant, $a_0$ is the cosmological acceleration, $r_e$ is the classical radius of the electron; $c$ - speed of light in vacuum; $t_0 = r_e/c$, $m_e$ - electron mass, $D_0$ - large Weyl number, $t_{\text{pl}}$ - Planck time, $l_{\text{pl}}$ - Planck length, $m_{\text{pl}}$ - Planck mass.
These relations are equal to large numbers. Many coincidences of large numbers involving the cosmological constant $\Lambda$ make it possible to derive new cosmological equations for various combinations of cosmological parameters and fundamental physical constants.

4. Systems of algebraic equations of the Universe

Several systems of equations can be formed from the cosmological equations (Fig. 3). In Fig. 4 shows four systems of cosmological equations.

![Figure 4](image)

**Figure 4**
Systems of cosmological equations for calculating the parameters of the observed Universe. Where: $\alpha$ - fine-structure constant, $h$ - Planck constant, $M_0$ - mass of the observable Universe, $G$ - Newtonian constant of gravitation, $\Lambda$ - cosmological constant, $R_0$ - radius of the observable Universe, $A_0$ - cosmological acceleration, $r_e$ - classical electron radius; $c$ - speed of light in vacuum; $m_e$ - electron mass, $D_0$ - large number

5. Parameters of the Universe

In these systems of cosmological equations only the fundamental physical constants $h, r_e, G, c, \alpha, m_e$ are known quantities. The unknown cosmological parameters $M_0, R_0, \Lambda, A_0, T_0$ are easily found by solving the system of equations. All
the given systems of cosmological equations (Fig. 4) give the same values of the Universe parameters (Fig. 5).

![Figure 5](image)

**Identical values of the Universe parameters obtained from different systems of algebraic equations of the Universe.**

The values of the Universe parameters and formulas for their calculation are given in Fig. 6.

![Figure 6](image)

**Values of the Universe parameters and formulas for their calculation.**

6. Cosmological acceleration constant.

One of the solutions of the systems of algebraic equations of the universe is the value of the cosmological acceleration constant as one of the parameters of the universe:

\[
A_0 = 10.4922\ldots \times 10^{-10} \text{ ms}^{-2} \tag{2}
\]

This parameter of the universe is unknown in Newton's law of gravitation. For this reason, Newton's theory of
gravitation has limitations and does not apply to cosmology. Using Newton’s second law, we obtain the value of the cosmological force \( F_{\text{cos}} \), which acts on all bodies in the universe and causes the acceleration \( A_0 \).

\[
F_{\text{cos}} = mA_0 = m \cdot 10.4922 \ldots \cdot 10^{-10} N
\]  

(3)

The cosmological acceleration constant \( A_0 \) can be represented by various equivalent formulas, such as:

\[
A_0 = GM_0 \Lambda = H^2/\sqrt{\Lambda} = c^2/\sqrt{\Lambda} = c^2/r_e \alpha D_0.
\]  

(4)

Where: \( G \) - Newtonian constant of gravitation, \( M_0 \) - mass of the observable Universe, \( \alpha \) - fine-structure constant, \( \Lambda \) - cosmological constant, \( H \) - Hubble constant, \( r_e \) - classical electron radius; \( c \) - speed of light in vacuum.

7. Law of cosmological force

The cosmological acceleration constant \( A_0 \) allows us to derive equations for the cosmological force. Fig. 7 shows three equivalent formulas for the cosmological force:

\[
F_{\text{cos}} = mGM_0 \Lambda \quad (5)
\]

\[
F_{\text{cos}} = mH^2/\sqrt{\Lambda} \quad (6)
\]

\[
F_{\text{cos}} = mc^2/\sqrt{\Lambda} \quad (7)
\]

Equivalent formulas for the law of cosmological force. Where:

\( F_{\text{cos}} \) - cosmological force, \( G \) - gravitational constant, \( M_0 \) - mass of the Universe, \( H \) - Hubble parameter, \( m \) - mass of a body, \( c \) - speed of light in vacuum, \( \Lambda \) - cosmological constant.

Formulas (5) and (6) include two parameters of the Universe each, formula (7) includes one parameter of the Universe. Of the three equivalent formulas, equation (7) is the simplest and most mathematically perfect.
The law of cosmological force gives a linear dependence of the gravitational interaction force on the body mass \( m \). The coupling constant in the new law of gravitation is the cosmological constant \( \Lambda \).

The law of cosmological gravitational force of the Universe is represented by a simple formula, which is not inferior in simplicity and perfection to the formula of Newton’s law of gravitation. The cosmological constant \( \Lambda \) fulfills the role of the coupling constant in the law of cosmological force. This determines the name of the law. The law shows with what force a body interacts with the mass of the Universe distributed in space. The law of cosmological force shows that any body of mass \( m \) is affected by the cosmological force of the Universe proportional to the mass of the body \( m \). The status of the cosmological constant \( \Lambda \) in the law of cosmological force is not less significant than the status of the constant \( G \) in Newton’s law.

Combining Newton’s law of gravitation and the law of cosmological force give a new Law of universal gravitation. The resulting force of gravitational interaction is defined as a vector sum of two forces: the Newtonian force and the cosmological force.

8. Law of universal gravitation

The law of cosmological force allows to present the law of universal gravitation in a new form. The law of universal gravitation should include not only the Newtonian force of interaction between two bodies, but also an additional cosmological force.

Fig. 9 shows the equivalent formulas of the law of universal gravitation.

![Figure 9: Equivalent formulas of the law of universal gravitation.](image)

The most simple and mathematically perfect is the following form of the Law of universal gravitation:

\[
\begin{align*}
F_s &= \frac{GmM}{r^2} \\
F_{cos} &= \frac{mM}{r^2} \\
F_{cos} &= \frac{me^2}{\sqrt{\Lambda}}
\end{align*}
\]
The law of universal gravitation contains two components. The first formula in the law of universal gravitation is Newton's well-known law of gravitation, and the second formula is the new law of cosmological force. Together the two laws of gravitation give the law of universal gravitation. The coupling constants in the law of universal gravitation are two constants: the gravitational constant $G$ and the cosmological constant $\Lambda$.

The contribution of the Newtonian force and the cosmological force to the total gravitational force depends on the distance and mass of the interacting bodies. For small masses and small distances, Newton's law of gravitation prevails. The fraction of the cosmological force for small masses is much smaller than the Newtonian force. The reason is the small value of the cosmological constant $\Lambda$ ($\Lambda = 1.36285... \times 10^{-52}$ m$^{-2}$). As the mass of the interacting bodies increases and the distance increases, the Newton force's fraction of the law of universal gravitation decreases and the cosmological force's fraction increases. The total gravitational force acting on a body is a vector sum of two forces: the Newton force and the cosmological gravitational force of the Universe.

9. Planck's constant and Planck units in classical gravitation.

The system of cosmological equations (Fig. 4-c.) shows that the parameters of the Universe are related to the Planck constant. A consequence of this is that all parameters of the universe can be represented using Planck units. From the coincidence of large numbers (Fig. 3) such equations are obtained:

$$M_U = m_p (\sqrt{\alpha D_0})^3$$

(8)
Formulas (8) – (12) give the same values of the Universe parameters as the systems of cosmological equations (Fig.11).

Values of the Universe parameters obtained from Planck units

The connection of the parameters of the Universe with Planck’s constant and with Planck units allows us to represent the Law of universal gravitation in quantum form.

10. Quantum law of universal gravitation.

All three equivalent formulas of the Law of universal gravitation (Fig. 9) can be represented in quantum form (Fig. 12):
It is possible that classical gravitation is not so far from quantum gravitation as it is commonly believed.

11. The theoretical limit of the cosmological force is equal to the Planck force $c^4/G$.

The study of the equivalent equations of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M} F_{Cos} = \lim_{m \to M} mc^3 \sqrt{\Lambda} = 1.21025 \times 10^{44} N = \frac{c^4}{G}$$

(13)

$$\lim_{m \to M} \frac{mH^2}{\sqrt{\Lambda}} = 1.21025 \times 10^{44} N = \frac{c^4}{G}$$

(14)

$$\lim_{m \to M} mGmU \Lambda = 1.21025 \times 10^{44} N = \frac{c^4}{G}$$

(15)

The theoretical limit of the cosmological force at $m \to M_0$ reaches the enormous value $c^4/G = 1.21025 \times 10^{44}$ N.

12. The coincidence of the cosmological acceleration $A_0 = 10.4922... \times 10^{-10}$ ms$^{-2}$ with the Pioneer anomaly.

The cosmological force for small values of masses is much smaller than the Newtonian force. Therefore, for small values of masses it is not pronounced and can be masked by effects of a non-gravitational nature. The unknown force was first experimentally detected in the Pioneer effect [6, 7, 8]. The Pioneers effect still has no convincing explanation. An attempt has been made to explain the effect by thermal recoil [9].

The new force that follows from the cosmological force law surprisingly turned out to be close to the Pioneer anomaly, which casts doubt on the thermal nature of the Pioneer effect. The significance of the cosmological acceleration that follows from the cosmological force law:

$$A_0 = c^2 \sqrt{\Lambda} = 10.4922... \times 10^{-10} m/s^2$$

(16)

The significance of the cosmological force:
\[ F_{\text{cos}} = m \cdot (10.4922 \cdot 10^{-10}) N \]  

(17)

Value of unknown force found in the pioneer effect:

\[ F_{\text{Pioneer}} = m \cdot (8.74 \pm 1.33) \cdot 10^{-10} N \]  

(18)

In addition to the Pioneer-10 and Pioneer-11 experiment, there is anomalous acceleration data from Galileo and Ulysses [10 - 13].

The value of the unknown force for Galileo:

\[ F_{\text{Galileo}} = m \cdot (8 \pm 3) \cdot 10^{-10} N \]  

(19)

Value of unknown force for Ulysses:

\[ F_{\text{Ulysses}} = m \cdot (12 \pm 3) \cdot 10^{-10} N \]  

(20)

The value of the cosmological force \( F = m(10.4922... \times 10^{-10}) N \) is very close to the experimental values \( F = m((8.74 \pm 1.33) \times 10^{-10}) N, F = m((8\pm3) \times 10^{-10}) N, F = m((12\pm3) \times 10^{-10}) N \). The coincidence of the force values casts doubt on the explanation of the pioneer anomaly by the temperature effect. The law of cosmological force points to the gravitational nature of the Pioneer Anomaly. The gravitational nature of the Pioneer anomaly was also pointed out by Hasmukh K. Tank in a study of critical-acceleration of MOND [14].

13. Conclusion

Newton's classical theory of gravitation was incomplete because of the absence in it of the cosmological component of the gravitational interaction. To complete it, the law of cosmological gravitational force was proposed in addition to Newton's law of gravitation. The new law shows that an additional cosmological force acts on any body in the Universe, except Newton's force. The new force is proportional to the mass of the body. The formula of the law of cosmological gravitational force is as follows:
The coupling constant in the law of cosmological force is the cosmological constant $\Lambda$. The law of cosmological force has no limitations of point masses and has no limitations of the law of inverse squares. On small scales, the cosmological force is much smaller than Newton's force. As the mass of a body increases, the cosmological force increases linearly. The cosmological force on the scale of the Universe is huge and tends to the theoretical limit equal to the Planck force $c^4/G = 1.21025 \times 10^{44}$ N.

With the appearance of the law of cosmological force, the law of universal gravitation takes a new form:

$$F_{\text{uni}} = G \frac{Mm}{r^2}$$

$$F_{\text{cos}} = mc^2 \sqrt{\Lambda}$$

The law of universal gravitation consists of two laws of force: Newton's law of gravitation and the law of cosmological force. The Newtonian gravitational force and the additional cosmological gravitational force of the Universe proportional to the mass of the body act on the body. The coupling constants in the new law of universal gravitation are two fundamental constants: the gravitational constant $G$ and the cosmological constant $\Lambda$.

In gravitational interaction there is always a cosmological force as an additional force to Newton's gravitational force. The new law of universal gravitation shows that at small distances the greatest contribution to the gravitational interaction is made by Newton's force. For large masses and distances on the scale of the Universe, the cosmological force makes a significant contribution to the gravitational interaction. As the mass of interacting bodies increases and the distance increases, the share of the Newton force decreases and the share of the cosmological force increases.

Thus, the law of cosmological force filled the missing link in Newton's classical gravitation. The law of cosmological force gives a force that acts beyond the
applicability of Newton's law. Newton's classical theory of gravitation becomes complete in the new law of universal gravitation. The law of universal gravitation in a completed form contains two partial laws of gravitation: Newton's law of gravitation and the law of cosmological force. The total force of gravitational interaction is a vector sum of two forces: Newton's force and the cosmological gravitational force of the Universe.

References: