METHOD OF CONVERTING Z-NUMBER TO CLASSIC FUZZY NUMBER

Abstract. Human beings have an amazing ability to make intelligent decisions based on inaccurate, unclear, or incomplete information. Forming such an opportunity is, in a sense, a difficult task. The proposed concept provides great opportunities for describing human knowledge and can be used in decision making in the process of working with fuzzy information. This article shows that the issue of converting a Z-number into a classical fuzzy number is very relevant. The article also presents a method for converting a Z-number to a fuzzy number using classical fuzzy set theory.

Keywords. Fuzzy set theory, Z-number theory, membership function, fuzzification, defuzzification.

The method of converting "Z-number to regular fuzzy number" is performed according to the algorithm described below.

Suppose the number \( Z = (A, B) \). The right side is reliability and the left side is limitations, \( A = \{(x, \mu_A(x)) | x \in [0,1]\} \) and \( B = \{(x, \mu_B(x)) | x \in [0,1]\} \), \( \mu_A(x) \) are trapezoidal membership functions, \( \mu_B(x) \) is triangle membership function. Here \( A = (a_1, a_2, a_3) \) and \( B = (b_1, b_2, b_3) \) are fuzzy numbers [5].

1) we convert second part (reliability) into a certain number.
where ∫ is algebraic integral.

2) The weight of the second part (reliability) is added to the first part (limitation). The weighted number Z can be expressed as follows

\[ Z'' = \{ (x, \mu_{A'}(x)) \mid \mu_{A'}(x) = \alpha \mu_A(x), x \in [0,1] \}, \]

\[ E_{A'}(x) = \alpha E_A(x), \quad x \in X, \]

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\[ E_{A'}(x) = \int_x x \mu_{A'}(x) dx = \int_x \alpha x \mu_A(x) dx = \alpha \int_x x \mu_A(x) dx = \alpha E_A(x). \]

According to a research [6, 7], information about two fuzzy numbers is reduced to one by integrating the second fuzzy number into the first fuzzy number. Valuation the degree of integration (weight) of a fuzzy number is expressed as follows:

\[ \alpha = \frac{1}{6} (b_1 + 4 \times b_2 + b_3). \]

Then the weight of the second part can be added to the first part, where the suspended Z-number can be given as follows:

\[ Z'' = (a_1, a_2, a_3; \alpha). \]

At the final stage, the suspended Z-number becomes a classic fuzzy number [3,4]:

\[ Z' = (a_1 \times \sqrt{\alpha}, a_2, a_1 \times \sqrt{\alpha}, a_1, a_1 \times \sqrt{\alpha}; 1). \]

1) Let the membership function \( \mu_{A'}(x) \) of a fuzzy number A be expressed in the following form:

\[ \mu_{A'}(x, a, b, c) = \begin{cases} 
\mu_A(x, a, b), & x \leq b, \\
1, & b \leq x \leq c, \\
1 - \mu_A(x, c + b - a), & x \geq c.
\end{cases} \]

Chen and Xie [8, 9, 10] proposed a mean-level integral representation to describe a generalized fuzzy number. S. Muruganandam later described the generalized fuzzy number.

Let \( L' \) and \( R' \) be the inverse functions of \( L \) and \( R \), respectively, \( A = (x, a, b, c : w) \) is a generalized fuzzy number with weight \( w \), its average \( h \)-level is
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\[ h \left[ L^1(h) + R^1(h) \right] / 2. \]

In this case, the default \( k \) value for the cumulative mean of the generalized fuzzy number based on the cumulative value of the weighted average \( h \)-level is:

\[
k = \frac{1}{2} \int_{0}^{w} \frac{h \left[ L^1(h) - R^1(h) \right]}{2} dh;
\]

where \( h \)-level is located between the numbers 0 and \( w \), \( 0 < w \leq 1 \).

2) Fuzzy number \( A \) is a fuzzy triangular number, denoted as \((a,b,c)\), for any \( n \), its membership function \( \mu_{A}(x) \) is expressed as follows

\[
\mu(x) = \begin{cases} 
\frac{x-a}{b-a}, & \text{if } x \leq b, \\
\frac{x-c}{b-c}, & \text{if } b \leq x \leq c.
\end{cases}
\]

Respectively, \( L^1 \) and \( R^1 \) be the inverse functions of \( L \) and \( R \) are calculated as follows:

\[
L(h) = \left\{ x : \frac{x-a}{c-a} = \sqrt[n]{h} \right\} = \left\{ x - a = (c-a) \sqrt[n]{h} \right\}, L(h) = a + (c-a) \sqrt[n]{h};
\]

\[
R(h) = \left\{ x : \frac{b-x}{b-c} = \sqrt[n]{h} \right\} = \left\{ b - x = (b-c) \sqrt[n]{h} \right\}, R(h) = b - (b-c) \sqrt[n]{h};
\]

In this case, the default \( k \) value for the cumulative mean of the generalized unknown number based on the cumulative value of the weighted average \( h \)-level is:

\[
k = \frac{1}{2} \int_{0}^{1} \frac{h \left[ a + \sqrt[n]{h}(c-a) + b - \sqrt[n]{h}(b-c) \right]}{2} dh.
\]

\( A = (a,b,c) \) - The form of a generalized triangle fuzzy number is calculated by the general formula:

\[
k = \frac{1}{2} \int_{0}^{1} \frac{h dh}{\sqrt[n]{h}} = \frac{2na + 2nb + a + b + 4nc - 2na - 2nb}{4n + 2} = \frac{a + 4nc + b}{4n + 2}.
\]

3) We express the fuzzy number \( A \) in Gaussian form, where \( \mu_{A}(x) \) membership
function is given below:

$$\mu_A(x) = e^{-tx}. $$

The function $L^{-1}$ is the inverse function of the function $L$:

$$L(h) = -\frac{\ln h}{t}. $$

Based on the cumulative average h-level, the default $k$ for the mean cumulative representation of the generalized fuzzy number is:

$$k = -\frac{1}{2t} \int_0^1 h \ln h \, dh - \int_0^1 h \, dh. $$

After applying fractional integration $\left\{ u = \ln h, \ dv = h \, dh \right\}$, a general formula for the form of a fuzzy number is calculated, expressed in the following form:

$$k = -\frac{1}{2t} \left[ \frac{h^2}{2} \ln h - \frac{1}{2} \frac{h^3}{2} \right] = -\frac{1}{4t}. $$

i.e. $k = -\frac{1}{4t}$. 

The aforementioned first and second steps have been performed, algebraic integrals and levels of fuzzy numerical integration (weights) have been found.

The disadvantages of the proposed approach should also be noted. The disadvantage of this method can be considered a partial loss of the original data as a result of converting Z-numbers into classic fuzzy numbers. The second part of the numbers is converted to a fuzzy number, therefore, the uncertainty described as the value of the reliability constraints at values that can be assumed by the existing initial uncertainties is lost. If the conversion to precision in the fuzzy inference algorithm occurs at the final stage of inference, then in the case of using substituted Z-numbers, the transition to precision occurs ahead of time. Since the conversion of Z-numbers to $Z = (A, B)$ is a fuzzy number, this is the multiplication of the probability value in the parameters of the first component of $A$ by a coefficient that assumes that it is in
the form of odd numbers, this coefficient $k$ can be calculated according to formula

$$
\mu_a(x) = \begin{cases} 
0, & x \in (-\infty, a_1), \\
\frac{x-a_1}{a_2-a_1}, & x \in [a_1, a_2], \\
\frac{a_3-x}{a_3-a_2}, & x \in [a_2, a_3], \\
0, & x \in (a_3, +\infty).
\end{cases}
$$

$$
k = \frac{1}{6} (b_1 + 4b_2 + b_3)
$$

Thus, a fuzzy inference system, described by $Z$-numbers in fuzzy numbers, allows to obtain the resulting values, considering the probability estimate, comprising $Z$-numbers, using the above.

The presence of such an approach to the use of $Z$-numbers in the system of fuzzy inferences allows one to more accurately take into account the uncertainty when working with approximate, fuzzy information. It is safe to say that such a developed algorithm can be widely used with great success in solving various problems, both engineering and economic.

It is not always possible to isolate a solution by $Z$-numbers using a single fuzzy set that describes all the properties of the object of study. Methods for obtaining a solution based on $Z$-numbers have been developed using the special compactness of $Z$-information and $Z$-compactness.

References: